Mass limit for Dirac-type magnetic monopoles

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We calculate the contribution of virtual monopole–antimonopole pairs to the anomalous magnetic moment of the muon $(g-2)_\mu$. If strong coupling QED generates confinement of monopole–antimonopole pairs, as suggested by recent lattice calculations, most of the monopole search experiments would not have been capable of detecting monopoles. However, such confined systems contribute to radiative corrections. We conclude that the effective monopole mass has to be larger than 120 GeV.

Ever since their postulation magnetic monopoles have been the object of intensive experimental searches. So far no convincing monopole signal has been found (for a review see ref. [1]). Recent lattice calculations [2] show evidence that strong coupling QED provides a confining phase for coupling constants α larger than $\frac{1}{3}\pi$. As the monopole coupling

$$\frac{g^2}{4\pi} \geqslant \left(\frac{2\pi}{e}\right)^2 \frac{1}{4\pi} = \frac{1}{4\alpha} \approx 34 \tag{1}$$

is much larger than this value, at least low mass monopoles could condense to some confined monopole–antimonopole or multi-monopole–antimonopole object. Such a state would have vanishing total magnetic charge. It might, however, be produced in strong magnetic fields and was discussed in connection with the unexplained electron–positron coincidences observed in heavy ion experiments at GSI [3]. We do not want to refer to these speculations in detail here. Instead we merely point out that confined magnetic monopoles would have evaded detection in experiments searching for particles with open magnetic charge. In this letter we will show that it is possible nevertheless to rule out light magnetic monopoles, since they would generate contributions to radiative corrections like the anomalous magnetic moment of the muon (fig. 1).

A calculation of such a process requires a consistent quantization of the monopole field which we assume to be fermionic. A corresponding procedure is well known in the literature [4].

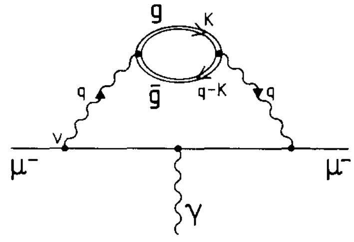


Fig. 1. The considered $(g-2)_\mu$ diagram.

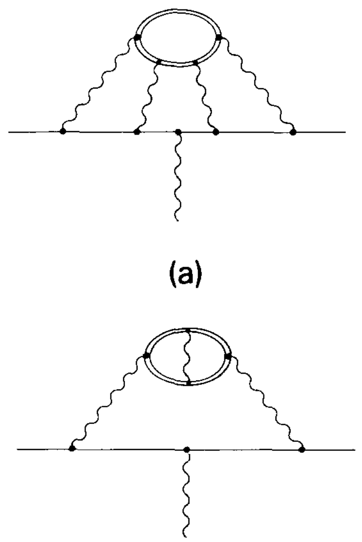
Since the coupling is strong ($g \gg e$), the contribution of the graph in fig. 1 is supposed to be large and thus imposes a lower bound on the monopole mass. This holds true only if the contributions do not vanish identically. On first sight this seems to be the case, as the magnetic charge couples to the dual field tensor. Thus the monopole–photon vertex has to contain an ϵ-tensor. As there are only three independent physical vectors, namely q_α, k_β and the polarization vector of the photon, any contraction of indices should therefore vanish.

The consistent quantization procedure shows, however, that there exists an additional vector which coincides with the direction of the Dirac string n_γ. The desired vertex then is proportional to $\epsilon_{\nu\alpha\beta\gamma} q^\alpha k^\beta n^\gamma$, which leads to a nonvanishing contribution and thus to a tight limit on the effective monopole mass. We follow ref. [4] and discuss the quantization only briefly.

We shall calculate the graph in fig. 1 only, although both $g^2/4\pi$ and $eg/4\pi$ are not small and higher order processes as depicted in fig. 2 are not necessarily negligible. Higher order contributions in the muon–monopole interaction (fig. 2a) involve a further power of the corresponding coupling constant $eg/4\pi=0.5$. This correction should be similar to the usual $g-2$ contribution [5]. Therefore the effective expansion parameter is given by $(eg/4\pi)(1/\pi)\approx 0.16$, which might allow for a perturbative expansion. Furthermore the graph (fig. 2a) should be suppressed by powers of m_μ/m_g relative to the graph in fig. 1.

The situation is different for higher order corrections in $g^2/4\pi$ (fig. 2b). The monopole–antimonopole interaction has definitely a nonperturbative character. In fact we follow ref. [2] and assume it to confine the monopoles. Thus the monopole–antimonopole state is a very complex object. Since we do not have information about the detailed structure of the confined system, we make the assumption that, for the virtual process discussed here, its effect is to replace the monopole mass by some effective quantity

$$m_g=\sqrt{m_0^2\left(\frac{O(1)}{R}\right)^2}\,, \tag{2}$$



(a)

(b)

Fig. 2. (a) Higher order muon–monopole interaction. (b) Higher order monopole–antimonopole interaction.

with the confinement radius R. Therefore graphs of higher order in $g^2/4\pi$ are supposed to be already absorbed in the treatment of virtual monopole–antimonopole states.

Starting with a purely magnetically charged fermion field χ coupled in the standard fashion to a vector gauge field C_μ, we introduce the dual field strength tensor $^*F_{\mu\nu}$. The action reads

$$\begin{aligned} S&=\int d^4x\,\mathscr{L} \\ &=\int d^4x[\bar\chi(i\partial_\mu\gamma^\mu-m_g)\chi-g\bar\chi\gamma^\mu\chi C_\mu \\ &\quad -\tfrac14(^*F_{\mu\nu})(^*F^{\mu\nu})]\,. \end{aligned} \tag{3}$$

This is related to the ordinary electric field strength tensor

$$^*F_{\mu\nu}=\tfrac12\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}\,, \tag{4}$$

$$^*F_{\mu\nu}=\partial_\mu C_\nu-\partial_\nu C_\mu\,, \tag{5}$$

with $\epsilon^{\mu\nu\alpha\beta}$ completely antisymmetric.

It is our goal now to eliminate the vector field C_μ. Since later we want to couple magnetic currents $j^\mu_{\rm mag}$ to the ordinary photon field A_μ, we need to know the structure of the interaction terms. From this point of view it is natural to relate the field tensor $^*F^{\mu\nu}$ to the ordinary tensor $F^{\mu\nu}$ through the simple form (4), rather than to establish a connection between the vectors C_μ and A_μ.

Introduction of a path-dependent U(1) gauge transformation

$$\chi(x)\Rightarrow\chi'(x)=\exp\left(ig\int_{p_x}dz^\mu\,C_\mu(z)\right)\chi(x) \tag{6}$$

allows for an elimination of the vector field C_μ at the expense of the path p_x which extends from infinity to the point x as we shall see [4,6].

The new fields χ' are invariant under U(1) transformations

$$\chi\Rightarrow\exp(-ig\Lambda)\,\chi\,, \tag{7}$$

$$\bar\chi\Rightarrow\bar\chi\exp(+ig\Lambda)\,, \tag{8}$$

$$C_\mu\Rightarrow C_\mu+\partial_\mu\Lambda\,, \tag{9}$$

but they depend on the path. According to (6) a variation of the path corresponds to a U(1) phase shift of the fermion field χ'. The desired degrees of freedom associated with the gauge field sector are the components of $*F_{\mu\nu}$. Consequently the covariant derivative D_μ has to be reexpressed in terms of the new variables $\bar{\chi}'$, χ', $*F^{\mu\nu}$ and p_x only:

$$\lim_{\epsilon^\mu\to 0} \epsilon^\mu D_\mu \chi'(x,p_x)$$

$$=\lim_{\epsilon^\mu\to 0}\bigg(\chi'(x^\mu+\epsilon^\mu,p_x)-\chi'(x^\mu,p_x)$$

$$+\mathrm{i}g\int d\eta\,\frac{\partial z^\mu}{\partial\eta}\frac{\partial z^\nu}{\partial x^\alpha}\epsilon^\alpha(*F_{\mu\nu})\cdot\chi'(x,p_x)\bigg). \qquad (10)$$

The path is parametrized in terms of η. The corresponding action does not depend on C_μ,

$$S=\int d^4x\,\mathscr{L}=\int d^4x$$

$$\times\bigg[\bar{\chi}'\bigg(\mathrm{i}\partial_\mu\gamma^\mu-g\int d\eta\,\frac{\partial z^\mu}{\partial\eta}\frac{\partial z^\nu}{\partial x^\alpha}(*F_{\mu\nu})\gamma^\alpha-m_{\mathrm{g}}\bigg)\chi'$$

$$-\tfrac{1}{4}(*F^{\mu\nu})(*F_{\mu\nu})\bigg]. \qquad (11)$$

Until now there is no explicit connection to the photon field A_μ and an electric current j^μ_{el} at all. This may be achieved in terms of an appropriate extension of the underlying lagrangian $\mathscr{L}$. One takes into account the usual inhomogeneous Maxwell equations

$$\partial_\mu F^{\mu\nu}=-\tfrac{1}{2}\partial_\mu\epsilon^{\mu\nu\alpha\beta}(*F_{\alpha\beta})=ej^\nu_{\mathrm{el}} \qquad (12)$$

that couple the gauge field sector, now described by its field tensor $*F^{\mu\nu}$, to the electric current j^ν_{el},

$$S\Rightarrow\int d^4x$$

$$\times\bigg[\bar{\chi}'\bigg(\mathrm{i}\partial_\mu\gamma^\mu-g\int d\eta\,\frac{\partial z^\mu}{\partial\eta}\frac{\partial z^\nu}{\partial x^\alpha}(*F_{\mu\nu})\gamma^\alpha-m_{\mathrm{g}}\bigg)\chi'$$

$$-\tfrac{1}{4}(*F_{\mu\nu})(*F^{\mu\nu})+\bar{\psi}(\mathrm{i}\partial_\mu\gamma^\mu-m_{\mathrm{e}})\psi$$

$$+A_\mu(\partial_\nu F^{\mu\nu}-e\bar{\psi}\gamma^\mu\psi)\bigg]. \qquad (13)$$

Here we have introduced an electrically charged fermion field ψ and consider in addition the photon field A_μ to be another degree of freedom.

In the next step it is necessary to dedude the Feynman rules of this theory. For this purpose we refer to the freedom of arbitrary path selection and restrict ourselves to a fixed spacelike p_x along the vector n^ν. Assuming that all fields vanish sufficiently at infinity, the desired monopole–photon vertex in momentum space is [4]

$$-\mathrm{i}g\frac{\gamma^\mu n^\nu q^\rho\epsilon_{\beta\mu\nu\rho}}{q\cdot n+\mathrm{i}\epsilon}. \qquad (14)$$

The dependence on the arbitrary vector n^ν relates to a violation of the Dirac veto. To some extent this dependence may be removed by setting the electric current j^μ_{el} to be zero on the string p_x.

Finally the squared amplitude for the process of monopole–antimonopole pair creation, depicted in fig. 3, reads

$$|U|^2=e^2g^2j^\beta_{\mathrm{mag}}j^{\alpha+}_{\mathrm{mag}}j^\mu_{\mathrm{el}}j^{\rho+}_{\mathrm{el}}\frac{g_{\mu\alpha}g_{\beta\rho}-g_{\mu\rho}g_{\beta\alpha}}{q^4}. \qquad (15)$$

This leads in lowest order to a cross section $\sigma(\mu^+\mu^-\to g\bar{g})$ which is up to constant factors identical to its purely electric counterpart $\sigma(\mu^+\mu^-\to e^+e^-)$. For our purpose the relevant cross section, with an off-shell photon γ, reads

$$\sigma(\gamma\Rightarrow g\bar{g})$$

$$=\frac{4\pi e^2g^2}{3\pi^2}\sqrt{\frac{s-4m_{\mathrm{g}}^2}{s}}\,\frac{1}{s}\bigg(1+\frac{2}{s}m_{\mathrm{g}}^2\bigg). \qquad (16)$$

Using a dispersion relation it is possible to compute the $(g-2)_\mu$ contributions of virtual $g\bar{g}$ pairs from this cross section just in the same manner as we did in ref. [7]. The $(g-2)_\mu$ contributions are given by

$$(g-2)_\mu$$

$$\Rightarrow\frac{1}{\pi^4}\int ds\,\sigma_{\gamma\to g\bar{g}}(s)\int_0^1 dx\,\frac{x^2(1-x)}{x^2+(s/m_\mu^2)(1-x)}. \qquad (17)$$

If the $g\bar{g}$-production threshold $s\geqslant 4m_{\mathrm{g}}^2$ is sufficiently large, (17) is approximately proportional to the ratio

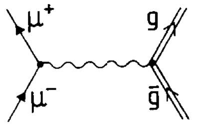


Fig. 3. The process of $g\bar{g}$ production through $\mu^+\mu^-$ annihilation in lowest order.

$(m_\mu/m_g)^2$. The anomalous magnetic moment of the muon is for that reason a more sensitive measure than the electronic one. We assume that the main properties of the confining interaction may be described in terms of a simple scalar potential, linearly increasing with the mutual separation r.

As discussed in ref. [7] our computation should be valid, if the confined $g\bar{g}$ system has a spatial extension $\langle r \rangle$ of roughly ten Compton wavelengths

$$\langle r \rangle > 10 \frac{1}{m_g} . \tag{18}$$

The corresponding dense discrete intrinsic $g\bar{g}$ spectrum may be interpreted as a discretized continuum, generated by imposing appropriate boundary conditions like the long range confinement. Referring then to the continuum, the contributions of virtual and confined $g\bar{g}$ pairs may be considered in an integration, rather than a summation over discrete states. The only adjustable parameter in order to control the $(g-2)_\mu$ contributions is now the effective monopole mass m_g.

Keeping all the problems and approximations of our calculations in mind, this approach should still give the correct order of magnitude for the desired $(g-2)_\mu$ contributions. Comparing (17) with experimental data [5], we find the lower limit for m_g to be

$$m_g > 120\ \text{GeV} . \tag{19}$$

This mass limit generates a limit on the spatial extension for which our calculations should be valid, which takes the value $\langle r \rangle \geqslant 10^{-3}$ fm at the lowest allowable mass m_g. The resulting mass constraint rules out that the sharp coincident e^+e^- lines observed in heavy ion collisions at the Coulomb barrier [3] can be explained by the production of $g\bar{g}$ pairs, as the required e^+e^- sum energy does not exceed 2 MeV.

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